Political Science Math Camp:
Problem Set 5

Due on Tuesday Aug 15th at 9:00 am

1. Consider the 2x2 matrix,
$$\begin{pmatrix} 1 & 2 \\ 2 & 4 \end{pmatrix}$$
   (a) What is the determinant of this matrix?
   (b) Find the inverse of the matrix; or, if you cannot find the inverse, say why not.
   (c) How many linearly independent rows does this matrix have? How many linearly independent columns?
   (d) What is the rank of the matrix?

2. Calculate the following derivatives:
   (a) $\frac{d}{dx}3x^{1/3}$
   (b) $\frac{d}{dx}(x^2+1)(x^3-1)$
   (c) $\frac{d}{dx}e^{x^2-3x+2}$
   (d) $\frac{d}{dx}\left[\frac{4x-12x^2}{x^3-4x^2}\right]^2$

3. Calculate the following integrals:
   (a) $\int(2x+12x^2)dx$
   (b) $\int 360t^6 dt$
   (c) $\int xe^{3x^2+1}dx$

4. The utility that a legislator obtains from a policy in a one-dimensional policy space is
$$U(x) = -a(x-i)^2$$
   where i is the legislators ideal point and a is the salience of the issue. This utility function formalizes the idea that the farther the policy x is from the legislators ideal policy i, the worse off the legislator is. The larger a is, the larger the effect that a given change in x has on the legislators utility. Suppose $a = 2$ and $i = 60$.
   (a) Prove that the legislators utility maximizing policy is $x = 60$.
   (b) Suppose that the constitution requires x to be in the range $x \in [70, 80]$. What policy in this range maximizes the legislators utility? Be sure to justify you answer.

5. The balance-of-power school in international relations argues that peace is most likely when there is a balance of power between states. By contrast, the preponderance-of-power school argues that that peace is least likely when there is a balance of power and is most likely when one side has a preponderance of power. Suppose that the analysis of a formal model yields the result that the probablilty of war in the model is

$$\pi = \frac{1}{4} - \left[p - \frac{1}{2}\right]^2$$

where π is the probability of war and p is the distribution of power between states 1 and 2. That is, p is the probability that 1 prevails in a war against 2. There is a balance of power when p = 1 / 2 . By contrast, 1 has a preponderance of power (in the extreme) when $p = 1$ and 2 has a preponderance of power when $p = 0$. Is the result of the formal analysis consistent with one of the two schools or neither? Be sure to explain your answer.

6. For the following functions, determine the interval over which the function is convex or concave (if at all), and identify any inflection points:

   (a) $f(x) = \frac{1}{x}$
   (b) $f(x) = x^2 + 4x + 8$
   (c) $f(x) = \frac{1}{1+x^2}$
   (d) $f(x) = log(x)$